An effective field theory for collinear and soft gluons: Heavy to light decays

Christian W. Bauer,[1] Sean Fleming,[2] Dan Pirjol,[1] and Iain W. Stewart[1]

[1]*Physics Department, University of California at San Diego, La Jolla, California 92093*
[2]*Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213*

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We construct the Lagrangian for an effective theory of highly energetic quarks with energy Q, interacting with collinear and soft gluons. This theory has two low energy scales, the transverse momentum of the collinear particles, $p_\perp$, and the scale $p_\perp^2/Q$. The heavy to light currents are matched onto operators in the effective theory at one loop and the renormalization group equations for the corresponding Wilson coefficients are solved. This running is used to sum Sudakov logarithms in inclusive $B\to X_s\gamma$ and $B\to X_u l\bar{\nu}$ decays. We also show that the interactions with collinear gluons preserve the relations for the soft part of the form factors for heavy-to-light decays found by Charles *et al.* [Phys. Rev. D **60**, 014001 (1999)], establishing these relations in the large energy limit of QCD.

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I. INTRODUCTION

The phenomenology of hadrons containing a single heavy quark is greatly simplified by the fact that nonperturbative hadronic physics can be parametrized by an expansion in $\Lambda_{\rm QCD}/m$, where m is the mass of the heavy quark. At lowest order, interactions are insensitive to the heavy quark mass and spin, leading to new spin-flavor symmetries [1]. These symmetries relate form factors for decays of one heavy hadron to another heavy hadron. In Refs. [1], [2] heavy quark effective theory (HQET) was constructed as a general framework in which to explore heavy quark physics. The effective theory allows a systematic treatment of $1/m$ corrections and makes the symmetries manifest. Inclusive decays of heavy hadrons involving large momentum transfer to the decay products can also be treated in HQET with the help of the operator product expansion (OPE) [3]. At leading order, the parton model results are recovered and nonperturbative corrections are parametrized by matrix elements of higher dimensional operators, suppressed by powers of $1/m$.

Decays of heavy hadrons to light hadrons cannot be treated exclusively with HQET unless the four-momentum of the light degrees of freedom are small compared to m. However, in regions of phase space where the light hadronic decay products have large energy $E\sim m$, a different expansion in powers of $1/E$ can be performed. In Ref. [4] Dugan and Grinstein used this approach to construct the large energy effective theory (LEET), which describes the interaction of very energetic quarks with soft gluons. However, LEET is missing an important degree of freedom, namely, collinear gluons, and does not reproduce the IR physics of QCD [5]. In Ref. [6] it was shown that an effective theory including both collinear and soft gluons correctly reproduces the infrared physics of QCD at one loop. This collinear-soft theory is needed between the scale E and an intermediate scale, below which collinear modes can be integrated out. For inclusive decays it was shown that the collinear-soft theory can be matched at the intermediate scale onto a theory containing only soft degrees of freedom.

The power counting in the collinear-soft theory is complicated by the presence of two low energy scales, which must be properly accounted for. These scales can be clearly seen by considering the momentum of a collinear quark. If the quark moves along the light-cone direction n^μ with momentum $Q\sim E\sim m$ then $p=(p^+,p^-,p_\perp)\sim Q(\lambda^2,1,\lambda)$, where λ is a small parameter. Thus $p_\perp\sim Q\lambda$ is the intermediate scale. With two low energy scales it is more appropriate to count powers of λ rather than powers of $1/Q$ [6]. This is analogous to nonrelativistic QCD (NRQCD) for bound states of two heavy quarks, where one counts powers of the velocity rather than powers of $1/m$ [7]. Constructing such an effective field theory bears some similarity to isolating momentum regimes using the method of regions [8] on full theory Feynman diagrams. There are, however, advantages to using an effective field theory approach over the method of regions, namely, it is straightforward to systematically include power corrections and it is possible to properly account for operator running, which sums Sudakov logarithms. In order to consistently go beyond leading order it is important to give a detailed construction of the effective field theory. This was not done in Ref. [6], and it is one of the main points of this paper.

The collinear-soft effective theory can be used to describe both inclusive and exclusive heavy-to-light decays. For inclusive decays this theory is valid in the regime where the phase space of the decay is restricted such that the final hadronic state is forced to have low invariant mass and large energy. This is the case for large electron energy or small hadronic invariant mass in semileptonic $B\to X_u l\bar{\nu}$ decays, and for large photon energy in $B\to X_s\gamma$ decays. The Sudakov logarithms that appear in the endpoint regions of these decays can be summed into the coefficient function of operators by running in the collinear-soft theory between the scale Q and $Q\lambda$, and then running a soft operator from $Q\lambda$ to $Q\lambda^2$. In Ref. [6] Sudakov logarithms at the endpoint of the photon energy spectrum in the decay $B\to X_s\gamma$ were summed in this manner. Here we sum Sudakov logarithms between Q and $Q\lambda$ for both $B\to X_s\gamma$ and $B\to X_u l\bar{\nu}$. In the ratio of large moments of these decay rates effects of physics below the intermediate scale cancel, and we reproduce previous calculations carried out using the factorization formalism [9,10].

It is also possible to apply the collinear-soft effective theory to exclusive heavy-to-light decays. The form factors for such transitions have contributions from the exchange of soft gluons with spectators (soft contributions), as well as from the exchange of hard gluons (so-called hard contributions). In this paper we will only consider the soft form factors, even though the two types of contributions are believed to be the same order in $1/m_b$ [11–13]. In Ref. [12] relations among the soft form factors were derived, and it was shown that only three independent functions are needed to describe heavy-to-light decays. However, these relations were obtained within the framework of LEET, which does not include collinear gluons. In this paper we show that the inclusion of collinear modes does not alter the soft form factor relations to leading order in λ. Since the collinear-soft effective theory reproduces the infrared physics of QCD at large energies, this establishes these soft form factor relations in the large energy limit of QCD.

In this paper we give a detailed construction of the collinear-soft effective theory and apply it to general heavy-to-light decays. In Sec. II the Lagrangian for collinear gluons and collinear quarks is constructed and the Feynman rules are given. The power counting for collinear gluons is formulated in a gauge invariant way. The collinear-soft effective theory does not have the same spin symmetry as LEET, but is still invariant under a helicity transformation. In Sec. III we construct the heavy-to-light currents in the effective theory at lowest order in λ. At this order the effective theory current couples to an arbitrary number of collinear gluons with a universal Wilson coefficient. The one-loop matching for the Wilson coefficients are then derived. In Sec. IV the renormalization group evolution of these coefficients are computed. Finally, in Sec. V we present two applications of this effective theory. First we sum Sudakov logarithms in the ratio of large moments of $B\to X_s\gamma$ and $B\to X_u l\bar{\nu}$ decay rates. Next we show that in the collinear-soft theory, only three independent soft form factors describe exclusive heavy-to-light decays, establishing these form factor relations in the large energy limit of QCD. The one-loop matching onto currents in the effective theory allows us to calculate the perturbative corrections to these form factor relations in an infrared safe manner. For the ratio of full theory form factors these hard corrections agree with Ref. [13].

II. THE EFFECTIVE THEORY

Decays of heavy hadrons to highly energetic light hadrons are most conveniently studied in the rest frame of the heavy hadron. In this reference frame the light particles move close to the light-cone direction n^μ and their dynamics is best described in terms of light-cone coordinates $p=(p^+,p^-,p_\perp)$, where $p^+=n\cdot p$, $p^-=\bar{n}\cdot p$. For motion in the z direction we take $n^\mu=(1,0,0,-1)$ and $\bar{n}^\mu=(1,0,0,1)$, so $\bar{n}\cdot n=2$. For large energies the different light cone components are widely separated, with $p^-\sim Q$ being large, while $p_\perp$ and p^+ are small. Taking the small parameter to be $\lambda\sim p_\perp/p^-$ we have

$$p^\mu=\bar{n}\cdot p\,\frac{n^\mu}{2}+(p_\perp)^\mu+n\cdot p\,\frac{\bar{n}^\mu}{2}=\mathcal{O}(\lambda^0)+\mathcal{O}(\lambda^1)+\mathcal{O}(\lambda^2), \qquad (1)$$

where we have used $p^+p^-\sim p_\perp^2\sim\lambda^2$ for fluctuations near the mass shell. The collinear quark can emit either a soft gluon or a gluon collinear to the large momentum direction and still stay near its mass shell. Collinear and soft gluons have light-cone momenta that scale like $k_c=Q(\lambda^2,1,\lambda)$ and $k_s=Q(\lambda^2,\lambda^2,\lambda^2)$, respectively. For scales above the typical off-shellness of the collinear degrees of freedom, $k_c^2\sim(Q\lambda)^2$, both gluon modes are required to correctly reproduce all the infrared physics of the full theory. This was described in [6], where it was shown that at a scale $\mu\sim Q$ QCD can be matched onto an effective theory that contains heavy quarks and light collinear quarks, as well as soft and collinear gluons.

The Lagrangian describing the interaction of collinear quarks with soft and collinear gluons can be obtained at tree level by expanding the full theory Lagrangian in powers of λ. We start from the QCD Lagrangian for massless quarks and gluons

$$\mathcal{L}_{\rm QCD}=\bar{\psi}i\not{D}\psi-\tfrac{1}{4}\,G_{\mu\nu}G^{\mu\nu}, \qquad (2)$$

where the covariant derivative is $D_\mu=\partial_\mu-igT^aA^a_\mu$, and $G_{\mu\nu}$ is the gluon field strength. We begin by removing the large momenta from the effective theory fields, similar to the construction of HQET [2]. In HQET there are two relevant momentum scales, the mass of the heavy quark m and $\Lambda_{\rm QCD}$. The scale m is separated from $\Lambda_{\rm QCD}$ by writing $p=mv+k$, where $v^2=1$ and the residual momentum $k\ll m$. The variable v becomes a label on the effective theory fields. Our case is slightly more complicated because there are three scales to consider. We split the momenta p by taking

$$p=\tilde{p}+k,\quad \text{where}\ \ \tilde{p}\equiv\frac{1}{2}(\bar{n}\cdot p)n+p_\perp. \qquad (3)$$

The ''large'' parts of the quark momentum $\bar{n}\cdot p\sim 1$ and $p_\perp\sim\lambda$, denoted by $\tilde{p}$, become a label on the effective theory field, while the residual momentum $k^\mu\sim\lambda^2$ is dynamical. This is analogous to NRQCD where there are also three relevant scales m, $m\beta$, and $m\beta^2$ (and $\beta\ll 1$ is the $q\bar{q}$ bound state velocity). In NRQCD the three scales can be separated [7] by writing $P=(m,\vec{0})+p+k$ where $p\sim m\beta$ and the residual momentum $k\sim m\beta^2$. In this case both the momenta of order m [i.e., $(1,\vec{0})$] and the momentum of order $m\beta$ are labels on the effective theory fields.

The large momenta $\tilde{p}$ are removed by defining a new field $\psi_{n,p}$ by

$$\psi(x)=\sum_{\tilde{p}}\,e^{-i\tilde{p}\cdot x}\psi_{n,p}(x)\,. \qquad (4)$$

A label p is given to the $\psi_{n,p}$ field, with the understanding that only the components $\bar{n}\cdot p$ and $p_\perp$ are true labels. Derivatives ∂^μ on the field $\psi_{n,p}(x)$ give order λ^2 contributions. For a particle moving along the n^μ direction, the four component

TABLE I. Power counting for the effective theory fields.

	Heavy quark	Collinear quark	Soft gluon	Collinear gluons		
Field	h_v	$\xi_{n,p}$	A_s^μ	$\bar{n}\cdot A_{n,q}$	$n\cdot A_{n,q}$	$A_{n,q}^\perp$
Scaling	λ^3	λ	λ^2	λ^0	λ^2	λ

field $\psi_{n,p}$ has two large components $\xi_{n,p}$ and two small components $\xi_{\bar{n},p}$. These components can be obtained from the field $\psi_{n,p}$ using projection operators

$$\xi_{n,p}=\frac{\not{n}\not{\bar{n}}}{4}\psi_{n,p}, \quad \xi_{\bar{n},p}=\frac{\not{\bar{n}}\not{n}}{4}\psi_{n,p}, \tag{5}$$

and satisfy the relations

$$\frac{\not{n}\not{\bar{n}}}{4}\xi_{n,p}=\xi_{n,p}, \quad \not{n}\xi_{n,p}=0,$$

$$\frac{\not{\bar{n}}\not{n}}{4}\xi_{\bar{n},p}=\xi_{\bar{n},p}, \quad \not{\bar{n}}\xi_{\bar{n},p}=0, \tag{6}$$

In terms of these fields the quark part of the Lagrangian in Eq. (2) becomes

$$\mathcal{L}=\sum_{\tilde{p},\tilde{p}'}\bigg[\bar{\xi}_{n,p'}\frac{\not{\bar{n}}}{2}(in\cdot D)\xi_{n,p}+\bar{\xi}_{\bar{n},p'}\frac{\not{n}}{2}(\bar{n}\cdot p+i\bar{n}\cdot D)\xi_{\bar{n},p} + \bar{\xi}_{n,p'}(\not{p}_\perp+i\not{D}_\perp)\xi_{\bar{n},p}+\bar{\xi}_{\bar{n},p'}(\not{p}_\perp+i\not{D}_\perp)\xi_{n,p}\bigg]. \tag{7}$$

Since the derivatives on the fermionic fields yield momenta of order $k\sim\lambda^2$, they are suppressed relative to the labels $\bar{n}\cdot p$ and $p_\perp$. Without the $\bar{n}\cdot D$ and $D_\perp$ derivatives, $\xi_{\bar{n},p}$ is not a dynamical field. Thus, we can eliminate $\xi_{\bar{n},p}$ at tree level by using the equation of motion

$$(\bar{n}\cdot p+\bar{n}\cdot iD)\xi_{\bar{n},p}=(\not{p}_\perp+i\not{D}_\perp)\frac{\not{\bar{n}}}{2}\xi_{n,p}. \tag{8}$$

This is similar to the approach taken in QCD quantized on the light cone [14] and in QCD in the infinite momentum frame [15], where two components of the fermion field are constrained auxiliary fields and are thus removed from the theory. Equations (7) and (8) result in a Lagrangian involving only the two components $\xi_{n,p}$,[1]

$$\mathcal{L}=\sum_{\tilde{p},\tilde{p}'}e^{-i(\tilde{p}-\tilde{p}')\cdot x}\bar{\xi}_{n,p'}\bigg[n\cdot iD+(\not{p}_\perp+i\not{D}_\perp)\frac{1}{\bar{n}\cdot p+\bar{n}\cdot iD} \times(\not{p}_\perp+i\not{D}_\perp)\bigg]\frac{\not{\bar{n}}}{2}\xi_{n,p}. \tag{9}$$

[1]Note that Eq. (9) still includes particle/antiparticle and the two spin degrees of freedom. However, on the light cone the spinor for a spin-up (down) particle is identical to that of the spin-up (down) antiparticle. See, for example, Ref. [15].

Here the summation extends over all distinct copies of the fields labeled by $\tilde{p},\tilde{p}'$. Note that the gluon field in D^μ includes collinear and soft parts, $A^\mu\to A_c^\mu+A_s^\mu$. The two types of gluons are distinguished by the length scales over which they fluctuate. Fluctuations of the collinear gluon fields A_c^μ are characterized by the scale $q^2\sim\lambda^2$, while fluctuations of the soft gluon field A_s^μ are characterized by $k^2\sim\lambda^4$. Since the collinear gluon field has large momentum components $\tilde{q}\equiv(\bar{n}\cdot q,q_\perp)$, derivatives acting on these fields can still give order $\lambda^{0,1}$ contributions. To make this explicit we label the collinear gluon field by its large momentum components $\tilde{q}$, and extract the phase factor containing $\tilde{q}$ by redefining the field $A_c(x)\to e^{-i\tilde{q}\cdot x}A_{n,q}(x)$. Inserting this into Eq. (9) one finds

$$\mathcal{L}=\sum_{\tilde{p},\tilde{p}',\tilde{q}}e^{-i(\tilde{p}-\tilde{p}')\cdot x}\bar{\xi}_{n,p'}\bigg[n\cdot iD+ge^{-i\tilde{q}\cdot x}n\cdot A_{n,q}+(\not{p}_\perp+i\not{D}_\perp + ge^{-i\tilde{q}\cdot x}\not{A}_{n,q}^\perp)\frac{1}{\bar{n}\cdot p+\bar{n}\cdot iD+ge^{-i\tilde{q}\cdot x}\bar{n}\cdot A_{n,q}}(\not{p}_\perp+i\not{D}_\perp + ge^{-i\tilde{q}\cdot x}\not{A}_{n,q}^\perp)\bigg]\frac{\not{\bar{n}}}{2}\xi_{n,p}. \tag{10}$$

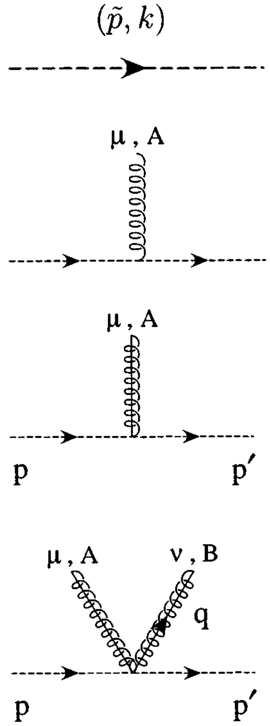
Here the covariant derivative is defined to only involve soft gluons.

Finally, we expand Eq. (10) in powers of λ. To simplify the power counting we follow the procedure [16] of moving all the dependence on λ into the interaction terms of the action to make the kinetic terms of order λ^0. This is done by assigning a λ scaling to the effective theory fields as given in Table I. The power counting in Table I gives an order one kinetic term for collinear gluons in an arbitrary gauge. In generalized covariant gauge

$$\int d^4x\, e^{ik\cdot x}\langle 0|TA_c^\mu(x)A_c^\nu(0)|0\rangle=\frac{-i}{k^2}\left(g^{\mu\nu}-\alpha\frac{k^\mu k^\nu}{k^2}\right) \tag{11}$$

and the scaling of the components on the right- and left-hand side of this equation agree.[2] With this power counting all interactions scale as λ^n with $n\geqslant 0$. Expanding Eq. (10) to order λ^0 gives

[2]We have chosen a different counting for the collinear gluon fields than Ref. [6] (where $A_c^\mu\sim\lambda$). In Feynman gauge there is the freedom to choose any scaling with $A_c^+A_c^-\sim\lambda^2$ (including the choice as in Ref. [6]). The choice in Table I is preferred since A_c^μ scales the same way as a collinear momentum and there are no interactions that scale as $1/\lambda$.



$$= i\,\frac{\not{\bar n}}{2}\,\frac{\bar n\cdot p}{n\cdot k\,\bar n\cdot p + p_\perp^2 + i\epsilon}$$

$$= ig T^A\, n_\mu\, \frac{\not{\bar n}}{2}$$

$$= ig T^A \left[n_\mu + \frac{\gamma_\mu^\perp \not{p}_\perp}{\bar n\cdot p} + \frac{\not{p}'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not{p}'_\perp \not{p}_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \right] \frac{\not{\bar n}}{2}$$

$$= \frac{ig^2 T^A T^B}{2\,\bar n\cdot(p-q)} \left[\gamma_\mu^\perp \gamma_\nu^\perp - \frac{\gamma_\mu^\perp \not{p}_\perp}{\bar n\cdot p}\bar n_\nu - \frac{\not{p}'_\perp \gamma_\nu^\perp}{\bar n\cdot p'}\bar n_\mu + \frac{\not{p}'_\perp \not{p}_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\not{\bar n}}{2}$$
$$+ \frac{ig^2 T^B T^A}{2\,\bar n\cdot(q+p')} \left[\gamma_\nu^\perp \gamma_\mu^\perp - \frac{\gamma_\nu^\perp \not{p}_\perp}{\bar n\cdot p}\bar n_\mu - \frac{\not{p}'_\perp \gamma_\mu^\perp}{\bar n\cdot p'}\bar n_\nu + \frac{\not{p}'_\perp \not{p}_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\not{\bar n}}{2}$$

FIG. 1. Order λ^0 Feynman rules: collinear quark propagator with label $\tilde p$ and residual momentum k, and collinear quark interactions with one soft gluon, one collinear gluon, and two collinear gluons, respectively.

$$\mathcal{L}_{cs} = \bar\xi_{n,p}\left(n\cdot iD + \frac{p_\perp^2}{\bar n\cdot p}\right)\frac{\not{\bar n}}{2}\xi_{n,p} + \bar\xi_{n,p+q}\Bigg[g n\cdot A_{n,q} + g\not{A}^\perp_{n,q}\frac{\not{p}_\perp}{\bar n\cdot p} + \frac{\not{p}_\perp + \not{q}_\perp}{\bar n\cdot (p+q)} g\not{A}^\perp_{n,q} - \frac{\not{p}_\perp + \not{q}_\perp}{\bar n\cdot(p+q)} g\bar n\cdot A_{n,q}\frac{\not{p}_\perp}{\bar n\cdot p}\Bigg]\frac{\not{\bar n}}{2}\xi_{n,p} + \cdots + \mathcal{O}(\lambda). \tag{12}$$

Summation over the labels $\tilde p, \tilde q$ is understood implicitly. The ellipsis denotes terms of the same order in the power counting with two or more collinear gluon fields, and arise because we expanded Eq. (10) in powers of gA_c to obtain the above expression. This expansion was necessary to move the collinear gluon phase factor appearing in the denominator of Eq. (10) into the numerator. This allowed us to remove the large momentum $\tilde q$ from the Lagrangian so that all covariant derivatives were truly of $\mathcal{O}(\lambda^2)$. The method for including terms of higher order in λ should be obvious from our derivation. The first few Feynman rules that follow from the λ^0 terms in $\mathcal{L}_{cs}$ are shown in Fig. 1.

The first term in Eq. (12) gives the propagator for the collinear quarks, which does not change depending on whether it interacts with soft or collinear gluons. This is distinct from the situation in the method of regions [8], where one must determine the propagator on a case by case basis. The interaction with a soft gluon is obtained from the covariant derivative term in Eq. (12). Also shown in Fig. 1 are the interactions with one and two collinear gluons. The collinear gluon interactions are label changing unlike the interaction involving soft gluons. Since $\bar n\cdot A_{n,q} \sim \bar n\cdot p$, Eq. (12) includes interactions of a collinear quark with an arbitrary number of collinear gluons. In Fig. 1 only interactions through $\mathcal{O}(g^2)$ are shown. Note that in the light-cone gauge $\bar n\cdot A_{n,q} = 0$ these Feynman rules are the complete set, since interactions of a collinear quark with three or more collinear gluons vanish. In this gauge similar Feynman rules for collinear gluons have been obtained in the framework of light-cone QCD [17]. However, the Feynman rules in Fig. 1 can be used in any gauge.

The LEET Lagrangian corresponds to the $\bar\xi_{n,p}(\not{\bar n}/2) n\cdot iD\xi_{n,p}$ term in Eq. (12) and is invariant under a SU(2) symmetry [4,12] with generators $S^1 = (\gamma^0\Sigma^1)/2$, $S^2 = (\gamma^0\Sigma^2)/2$, and $S^3 = \Sigma^3/2$ where Σ^i are the standard rotation generators. The collinear soft Lagrangian $\mathcal{L}_{cs}$ has less symmetry than LEET because terms with $\gamma_\perp^1\gamma_\perp^2$ violate the transformations generated by S^1 and S^2. However, $\mathcal{L}_{cs}$ is still invariant under a $U(1)$, namely, the helicity transformations generated by S^3. Since $S^3 = \gamma^5(1/2 - \not{\bar n}\not{n}/4)$ and $\not{n}\xi_{n,p} = 0$ the helicity transformation also corresponds to the chiral transformation generated by $\gamma^5/2$.

To complete the construction of the effective theory we have to include heavy quarks. This can be done by adding the usual HQET Lagrangian for the heavy quark field h_v,

$$\mathcal{L}_{HQET} = \bar h_v iv\cdot D h_v . \tag{13}$$

The covariant derivative in Eq. (13) contains only the soft gluon field because the heavy quark field does not couple to collinear gluons [6]. This is discussed in more detail in the next section.

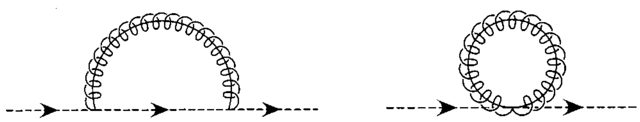


FIG. 2. Order $\alpha_s\lambda^0$ self-energy diagrams for a collinear quark.

As a simple application of the Feynman rules consider the order λ^0 diagrams for the self-energy of a collinear quark shown in Fig. 2. The tadpole diagram vanishes in dimensional regularization. In the Feynman gauge the remaining diagram gives

$$i\Sigma_c(p)=g^2C_F\frac{\bar{n}\!\!\!/}{2}\int\frac{d^dl}{(2\pi)^d}\Bigg\{(n\cdot\bar{n})\frac{p_\perp^2+p\!\!\!/_\perp l\!\!\!/_\perp}{\bar{n}\cdot p(p+l)^2l^2}+(n\cdot\bar{n})\frac{p_\perp^2+l\!\!\!/_\perp p\!\!\!/_\perp}{\bar{n}\cdot p(p+l)^2l^2}+2(d-4)\frac{p_\perp^2+l_\perp\cdot p_\perp}{\bar{n}\cdot p(p+l)^2l^2}-(d-2)\left(\frac{(p_\perp+l_\perp)^2}{[\bar{n}\cdot(p+l)]^2}+\frac{p_\perp^2}{[\bar{n}\cdot p]^2}\right)\frac{\bar{n}\cdot(p+l)}{(p+l)^2l^2}\Bigg\}. \tag{14}$$

Here sums over the labels $\bar{n}\cdot l$ and $l_\perp$ were combined with the integrals over residual momenta to give the full d^dl measure (cf., Ref. [7]). The first two terms in Eq. (14) correspond to the $(\mu,\nu)=(+,-)$ and $(-,+)$ polarizations of the exchanged gluon, and the last line to the $(\perp,\perp)$ contribution, respectively. Computing the loop integrals one finds

$$i\Sigma_{+-}(p)+i\Sigma_{-+}(p)=\frac{i\alpha_sC_F}{4\pi}\frac{\bar{n}\!\!\!/}{2}\Gamma(\epsilon)\frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)}\frac{2p_\perp^2}{\bar{n}\cdot p}\left(\frac{-p^2}{e^{\gamma_E}\mu^2}\right)^{-\epsilon}, \tag{15}$$

$$i\Sigma_{\perp\perp}(p)=-\frac{i\alpha_sC_F}{4\pi}\frac{\bar{n}\!\!\!/}{2}\Gamma(\epsilon)\frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)}\left\{(1+\epsilon)\frac{p_\perp^2}{\bar{n}\cdot p}-(1-\epsilon)n\cdot p\right\}\left(\frac{-p^2}{e^{\gamma_E}\mu^2}\right)^{-\epsilon}.$$

Here (and in the rest of the paper) we use modified minimal subtraction ($\overline{\rm MS}$) and therefore redefined $\mu^2\to\mu^2e^{\gamma_E}/(4\pi)$. The sum has precisely the form of the inverse collinear quark propagator in Fig. 1,

$$\Sigma_c(p)=\frac{\alpha_sC_F}{4\pi}\frac{\bar{n}\!\!\!/}{2}(1-\epsilon)\Gamma(\epsilon)\frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)}\frac{p^2}{\bar{n}\cdot p}\left(\frac{-p^2}{e^{\gamma_E}\mu^2}\right)^{-\epsilon}. \tag{16}$$

The ultraviolet divergence in this expression is removed by on-shell wave function renormalization of the effective theory field $\xi_{n,p}$,

$$Z_\xi=1-\frac{\alpha_sC_F}{4\pi}\left[\frac{1}{\epsilon}-\ln\left(\frac{-p^2}{\mu^2}\right)+1\right]. \tag{17}$$

Z_ξ coincides with the renormalization of the quark field in QCD. This is expected [6] since without currents or soft effects the collinear quark Lagrangian simply describes QCD in a particular frame. The utility of the two component formalism in Eq. (12) will become evident in the next section where heavy-to-light currents are discussed.

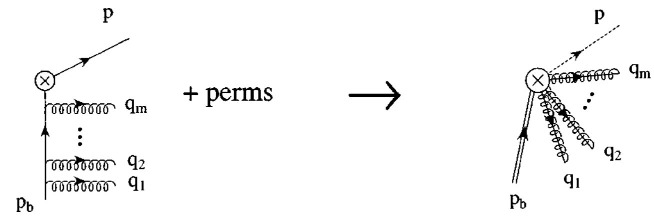


FIG. 3. Matching for the order λ^0 Feynman rule for the heavy-to-light current with n collinear gluons. All permutations of crossed gluon lines are included on the left.

III. MATCHING THE HEAVY-TO-LIGHT CURRENTS

At a scale $\mu\sim Q$ the weak Hamiltonian has heavy-to-light semileptonic or radiative operators of the form [18]

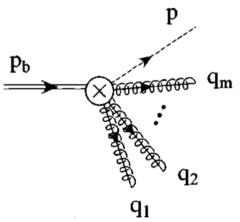
$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}VC^{\rm full}(\mu)J_{\rm had}\,J, \tag{18}$$

where V is the Cabibbo-Kobayashi-Maskawa (CKM) factor, J is a nonhadronic current, and the Wilson coefficients $C^{\rm full}(\mu)$ have been run from the scale $\mu=m_W$ down to m_b. In Eq. (18), the hadronic currents are of the form $J_{\rm had}=\bar{q}\Gamma b$ and we will consider $\Gamma=\{1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu},\sigma_{\mu\nu}\gamma_5\}$. We choose this overcomplete basis to simplify the treatment of $b\to s\gamma$. Below the scale $Q\sim\bar{n}\cdot p$ the hadronic current can be matched onto currents in the collinear-soft effective theory. This introduces a new set of Wilson coefficients $C_i(\mu)$. In this section the one-loop matching for these new coefficients will be performed at $\mu=m_b$, while the running will be considered in Sec. IV. We could equally well match at $\mu=\bar{n}\cdot p$, but the difference is irrelevant since we treat $\bar{n}\cdot p\sim m_b$ and do not attempt to sum logarithms of the form $\ln(\bar{n}\cdot p/m_b)$.

Naively, one might expect that at lowest order the effective theory hadronic current is $J_{\rm had}^{\rm eft}=C(\mu)\bar{\xi}_{n,p}\Gamma h_v$. However, since the label $\bar{n}\cdot p\sim\lambda^0$, the effective theory Wilson coefficient can also be a function of $\bar{n}\cdot p$. Furthermore, an arbitrary number of fields $\bar{n}\cdot A_{n,q}\sim\lambda^0$ can be included without additional power suppression. At lowest order in λ the most general heavy-to-light current in the effective theory therefore has the form

$$J_{\rm had}^{\rm eft}=c_0(\bar{n}\cdot p,\mu)\bar{\xi}_{n,p}\Gamma h_v+c_1(\bar{n}\cdot p,\bar{n}\cdot q_1,\mu)\bar{\xi}_{n,p}(g\bar{n}\cdot A_{n,q_1})\Gamma h_v+c_2(\bar{n}\cdot p,\bar{n}\cdot q_1,\bar{n}\cdot q_2,\mu)\bar{\xi}_{n,p}(g\bar{n}\cdot A_{n,q_1})\times(g\bar{n}\cdot A_{n,q_2})\Gamma h_v+\cdots, \tag{19}$$

where the ellipsis stands for terms of the same order with more powers of $\bar{n}\cdot A_{n,q}$. The coefficients c_i may also depend on the choice of Γ. At the scale $\mu=m_b$ the c_i can be determined by the tree level matching calculation depicted in Fig. 3. On the left, the gluons with collinear momenta kick the b



$$= iC(\mu, \bar{n}\cdot P)\Gamma \frac{g^m}{m!} \sum_{\text{perms}} \frac{(\bar{n}^{\mu_m} T^{A_m})\cdots(\bar{n}^{\mu_1} T^{A_1})}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{m} q_i]}$$

FIG. 4. Order λ^0 Feynman rule for the effective theory heavy-to-light current with m collinear gluons. The sum is over permutations of $\{1,\ldots,m\}$ and the Wilson coefficient depends only on the sum of momenta in the jet, $P=p+\Sigma_{i=1}^{m} q_i$.

quark far off-shell, and integrating out these off-shell b quarks gives the effective theory operator on the right.

To perform the matching, first consider the simpler case of an Abelian gauge group. In this case calculating the full theory graph with m gluons in Fig. 3, expanding in powers of λ, and putting the result over a common denominator gives

$$c_m(\mu=m_b)=\frac{1}{m!}\prod_{i=1}^{m}\frac{1}{\bar{n}\cdot q_i}. \tag{20}$$

The factor of $1/m!$ is from the presence of m identical A_c fields at the same point.[3] Thus, we have the tree level result

$$J_{\text{had}}^{\text{eft}}\big|_{\mu=m_b}=\bar{\xi}_{n,p}\exp\left(\frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\Gamma h_v\,. \tag{21}$$

It is not immediately clear how this result is modified for $\mu<m_b$ since the infinite series of operators in Eq. (19) could each run differently. However, gauge invariance relates these operators, and only the sum of terms in Eq. (21) is gauge invariant. Under a collinear gauge transformation $\alpha(x)$, the field h_v is invariant since collinear gluons do not couple to heavy quarks. On the other hand, the collinear quark field transforms as $\xi_{n,p}\to e^{i\alpha(x)}\xi_{n,p}$. Thus, the operator $\bar{\xi}_{n,p}\Gamma h_v$ is not gauge invariant. However, it is straightforward to see that the operator in Eq. (21) is invariant, and this is done in Appendix A. It is found that

$$\exp\left(\frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\to\exp\left(\frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\exp[i\alpha(x)], \tag{22}$$

and the last exponential exactly cancels the transformation of $\bar{\xi}_{n,p}$. By gauge invariance the current therefore has to be of the form in Eq. (21) for an arbitrary scale μ. It is convenient to define a field that transforms as a singlet under a collinear gauge transformation

$$\chi_{n,P}=\exp\left(\frac{-g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\xi_{n,p}\,. \tag{23}$$

We will refer to $\chi_{n,P}$ as the jet field since it involves a collinear quark field plus an arbitrary number of collinear gluons moving in the n direction. The relevant label for the jet field is simply the sum of labels of the particles in the jet, $P=p+\Sigma q_i$. In terms of this field the leading order effective theory current for $Q\lambda<\mu<m_b$ has the form

$$J_{\text{had}}^{\text{eft}}=C_i(\mu,\bar{n}\cdot P)\bar{\chi}_{n,P}\Gamma h_v\,, \tag{24}$$

with a universal coefficient $C_i(\mu,\bar{n}\cdot P)$. The statement that the coefficient only depends on the total jet momentum P is nontrivial and is discussed further in Appendix B.

For a non-Abelian gauge group a similar gauge invariance argument applies, however the matching in Fig. 3 is more complicated. Equation (24) remains valid, but with a more complicated definition of the jet field. In momentum space we find

$$\chi_{n,P}=\sum_k\sum_{\text{perms}}\frac{(-g)^k}{k!}\times\left\{\frac{\bar{n}\cdot A_{\bar{n}q_1}\cdots\bar{n}\cdot A_{\bar{n}q_k}}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots\left[\bar{n}\cdot\sum_{i=1}^{k}q_i\right]}\right\}\xi_{n,p}\,, \tag{25}$$

where the permutation sum is over the indices $(1,2,\ldots,k)$. The Feynman rules that follow from Eqs. (24) and (25) are shown in Fig. 4. In position space the jet field takes the form of a path-ordered exponential

$$\chi_n(0)=P\exp\left[-ig\int_{-\infty}^{0}ds\,\bar{n}\cdot A^c(s\bar{n}^{\mu})\right]\xi_n(0), \tag{26}$$

where P denotes path ordering along the lightlike line collinear to $\bar{n}$.[4]

In the effective theory both heavy and light quarks are described by two component spinors, so there are only four heavy-to-light currents at leading order in λ. We choose the linearly independent set $[\bar{\chi}_{n,P}h_v]$, $[\bar{\chi}_{n,P}\gamma_5 h_v]$, and $[\bar{\chi}_{n,P}\gamma_\perp^{\mu}h_v]$, where $\gamma_\perp^{\mu}=\gamma^{\mu}-n^{\mu}\bar{n}\!\!\!/ /2-\bar{n}^{\mu}n\!\!\!/ /2$ has only two nonzero terms. The matching of the heavy to light currents $\bar{q}\Gamma b$ onto operators in the effective theory is

$$\bar{q}b\to C_1(\mu)[\bar{\chi}_{n,P}h_v],$$

$$\bar{q}\gamma_5 b\to C_2(\mu)[\bar{\chi}_{n,P}\gamma_5 h_v],$$

[3]Note that in Ref. [6] the Feynman rule with a single collinear gluon ($m=1$) has an additional $n\!\!\!/ A\!\!\!/_c/m_b$ term that did not contribute to the results there. With the power counting in the Table I this term is suppressed by a power of λ.

[4]Path-ordered exponentials are also introduced to sum up the couplings of soft gluons to a collinear jet, see Ref. [19].

$$\begin{aligned}
\bar{q}\gamma_\mu b &\to C_3(\mu)[\bar{\chi}_{n,P}\gamma_\mu^\perp h_v] \\
&\quad + \{C_4(\mu)n_\mu + C_5(\mu)v_\mu\}[\bar{\chi}_{n,P}h_v], \\
\bar{q}\gamma_\mu\gamma_5 b &\to C_6(\mu)i\epsilon^\perp_{\mu\nu}[\bar{\chi}_{n,P}\gamma_\perp^\nu h_v] \\
&\quad - \{C_7(\mu)n_\mu + C_8(\mu)v_\mu\}[\bar{\chi}_{n,P}\gamma_5 h_v], \qquad (27) \\
\bar{q}i\sigma_{\mu\nu}b &\to C_9(\mu)(n_\mu g_{\nu\lambda} - n_\nu g_{\mu\lambda})[\bar{\chi}_{n,P}\gamma_\perp^\lambda h_v] \\
&\quad + C_{10}(\mu) \\
&\quad \times i\epsilon^\perp_{\mu\nu}[\bar{\chi}_{n,P}\gamma_5 h_v] + C_{11}(\mu)(v_\mu n_\nu - v_\nu n_\mu) \\
&\quad \times [\bar{\chi}_{n,P}h_v] + C_{12}(\mu)(v_\mu g_{\nu\lambda} - v_\nu g_{\mu\lambda}) \\
&\quad \times [\bar{\chi}_{n,P}\gamma_\perp^\lambda h_v], \\
\bar{q}i\sigma_{\mu\nu}\gamma_5 b &\to -[C_9(\mu) + C_{12}(\mu)](in_\mu\epsilon^\perp_{\nu\lambda} - in_\nu\epsilon^\perp_{\mu\lambda}) \\
&\quad \times [\bar{\chi}_{n,P}\gamma_\perp^\lambda h_v] + C_{11}(\mu)i\epsilon^\perp_{\mu\nu}[\bar{\chi}_{n,P}h_v] \\
&\quad + C_{10}(\mu)(v_\mu n_\nu - v_\nu n_\mu)[\bar{\chi}_{n,P}\gamma_5 h_v] \\
&\quad + C_{12}(\mu)(iv_\mu\epsilon^\perp_{\nu\lambda} - iv_\nu\epsilon^\perp_{\mu\lambda})[\bar{\chi}_{n,P}\gamma_\perp^\lambda h_v],
\end{aligned}$$

where $\epsilon^\perp_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}v^\rho n^\sigma$ with $\epsilon_{0123} = -1$. From here on the dependence of the Wilson coefficients on $\bar{n}\cdot P$ will be suppressed. The relations in Eq. (27) are valid[5] to all orders in α_s and leading order in λ. At tree level the matching gives

$$C_{1,2,3,4,6,7,9,10,11}(m_b) = 1, \quad C_{5,8,12}(m_b) = 0. \qquad (28)$$

To match these coefficients at one loop, we calculate perturbative matrix elements in the full and effective theories. All the long distance physics is reproduced in the effective theory, and the difference between the two calculations determines the short distance Wilson coefficients. Since the Wilson coefficients are universal the matching can be performed for the simpler current $\bar{\xi}_{n,p}\Gamma h_v$ rather than the current $\bar{\chi}_{n,P}\Gamma h_v$. The calculation is most easily performed in pure dimensional regularization. The full theory matrix elements of the currents $\bar{q}\Gamma b$ between free quark states are obtained by evaluating the diagram in Fig. 5 and multiplying by the wave function and current renormalization factors. In $d = 4 - 2\epsilon$ dimensions the on-shell wave function renormalization constants for massive and massless quarks are

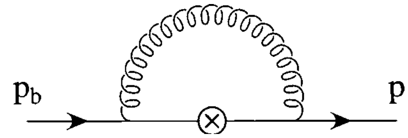


FIG. 5. Full theory one-loop diagram for matching the heavy-to-light current (denoted by ⊗). The incoming line is a massive quark and the outgoing line is massless.

$$Z_b = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{3}{\epsilon} + 3\ln\frac{m_b^2}{\mu^2} - 4\right), \quad Z_q = 1, \qquad (29)$$

and the renormalization constants for the scalar, pseudoscalar, vector, axial vector, tensor, and axial tensor currents are given by

$$Z_S = Z_P = 1 - \frac{3\alpha_s C_F}{4\pi\epsilon}, \quad Z_V = Z_A = 1,$$

$$Z_T = Z_{T_5} = 1 + \frac{\alpha_s C_F}{4\pi\epsilon}. \qquad (30)$$

The ultraviolet divergences in the Z's cancel the ultraviolet divergences in the diagram in Fig. 5, hence all remaining $1/\epsilon$ divergences are of infrared nature. The b quark and light quark are taken to have momenta p_b and p, respectively, and we define $q = p_b - p$. Letting γ_5 anticommute in d dimensions [the naive dimensional regularization (NDR) scheme], the final result for the matrix elements in the full theory is

$$\begin{aligned}
\langle q|\bar{q}\{1,\gamma_5\}b|b\rangle &= \Bigg\{1 - \frac{\alpha_s C_F}{4\pi}\Bigg[\frac{1}{\epsilon^2} + \frac{5}{2\epsilon} + \frac{\ln\left(\frac{\mu^2}{m_b^2}\right)}{\epsilon} - \frac{2\ln(1-\hat{q}^2)}{\epsilon} + \frac{1}{2}\ln^2\left(\frac{\mu^2}{m_b^2}\right) - \frac{1}{2}\ln\left(\frac{\mu^2}{m_b^2}\right) - 2\ln(1-\hat{q}^2)\ln\left(\frac{\mu^2}{m_b^2}\right) \\
&\quad + 2\ln^2(1-\hat{q}^2) - \frac{2\ln(1-\hat{q}^2)}{\hat{q}^2} + 2\,\mathrm{Li}_2(\hat{q}^2) + \frac{\pi^2}{12}\Bigg]\Bigg\}\bar{u}\{1,\gamma_5\}u, \\
\langle q|\bar{q}\{1,\gamma_5\}\gamma^\mu b|b\rangle &= \Bigg\{1 - \frac{\alpha_s C_F}{4\pi}\Bigg[\frac{1}{\epsilon^2} + \frac{5}{2\epsilon} + \frac{\ln\left(\frac{\mu^2}{m_b^2}\right)}{\epsilon} - \frac{2\ln(1-\hat{q}^2)}{\epsilon} + \frac{1}{2}\ln^2\left(\frac{\mu^2}{m_b^2}\right) + \frac{5}{2}\ln\left(\frac{\mu^2}{m_b^2}\right) - 2\ln(1-\hat{q}^2)\ln\left(\frac{\mu^2}{m_b^2}\right) \\
&\quad + 2\ln^2(1-\hat{q}^2) + \ln(1-\hat{q}^2)\left(\frac{1}{\hat{q}^2} - 3\right) + 2\,\mathrm{Li}_2(\hat{q}^2) + \frac{\pi^2}{12} + 6\Bigg]\Bigg\}\bar{u}\{1,\gamma_5\}\gamma^\mu u \\
&\quad + \frac{\alpha_s C_F}{4\pi}\left[\frac{4}{\hat{q}^2}\ln(1-\hat{q}^2) - \frac{2}{\hat{q}^2} - \frac{2}{\hat{q}^4}\ln(1-\hat{q}^2)\right]\hat{p}^\mu\bar{u}\{1,\gamma_5\}u
\end{aligned}$$

[5]An exception is the relation between the coefficients for $\bar{q}i\sigma_{\mu\nu}b$ and $\bar{q}i\sigma_{\mu\nu}\gamma_5 b$ that can change depending on how γ_5 is treated in d dimensions.

$$+\frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\hat{q}^2}-\frac{2}{\hat{q}^2}\ln(1-\hat{q}^2)+\frac{2}{\hat{q}^4}\ln(1-\hat{q}^2)\right]\hat{p}_b^\mu\overline{u}\{1,\gamma_5\}u,$$

$$\langle q|\overline{q}\{1,\gamma_5\}i\sigma^{\mu\nu}b|b\rangle=\left\{1-\frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon^2}+\frac{5}{2\epsilon}+\frac{\ln\left(\frac{\mu^2}{m_b^2}\right)}{\epsilon}-\frac{2\ln(1-\hat{q}^2)}{\epsilon}+\frac{1}{2}\ln^2\left(\frac{\mu^2}{m_b^2}\right)+\frac{7}{2}\ln\left(\frac{\mu^2}{m_b^2}\right)-2\ln(1-\hat{q}^2)\ln\left(\frac{\mu^2}{m_b^2}\right)\right.\right.$$
$$\left.\left.+2\ln^2(1-\hat{q}^2)+2\ln(1-\hat{q}^2)\left(\frac{1}{\hat{q}^2}-2\right)+2\,\mathrm{Li}_2(\hat{q}^2)+\frac{\pi^2}{12}+6\right]\right\}\overline{u}\{1,\gamma_5\}i\sigma^{\mu\nu}u$$
$$+\frac{\alpha_s C_F}{4\pi}\left[\frac{4}{\hat{q}^2}\ln(1-\hat{q}^2)\right]\overline{u}\{1,\gamma_5\}(\hat{p}^\mu\gamma^\nu-\hat{p}^\nu\gamma^\mu)u, \tag{31}$$

where the hat denotes momenta normalized with respect to m_b, so $\hat{q}=q/m_b$. This full theory result can be expanded in λ by noting that

$$\hat{q}^2=1-\overline{n}\cdot\hat{p}+\mathcal{O}(\lambda^2), \tag{32}$$

and that at lowest order we can expand the full theory spinors using Eqs. (27) and (28).

For the effective theory in pure dimensional regularization the final collinear quark is taken on-shell. For momentum labels $(\overline{n}\cdot p,p_\perp)$ this corresponds to choosing this quarks residual momentum k such that $\overline{n}\cdot p\, n\cdot k+p_\perp^2=0$. In this case all graphs in the effective theory are proportional to $1/\epsilon_{\rm UV}-1/\epsilon_{\rm IR}=0$. The ultraviolet divergences are canceled by effective theory counterterms, and all infrared divergences cancel in the difference between the full and effective theories. Thus, from Eq. (31) the Wilson coefficients at the scale $\mu=m_b$ are

$$C_{1,2}(m_b)=1-\frac{\alpha_s(m_b)C_F}{4\pi}\left\{2\ln^2(\overline{n}\cdot\hat{P})+2\,\mathrm{Li}_2(1-\overline{n}\cdot\hat{P})-\frac{2\ln(\overline{n}\cdot\hat{P})}{1-\overline{n}\cdot\hat{P}}+\frac{\pi^2}{12}\right\},$$

$$C_{3,6}(m_b)=1-\frac{\alpha_s(m_b)C_F}{4\pi}\left\{2\ln^2(\overline{n}\cdot\hat{P})+2\,\mathrm{Li}_2(1-\overline{n}\cdot\hat{P})+\ln(\overline{n}\cdot\hat{P})\left(\frac{3\overline{n}\cdot\hat{P}-2}{1-\overline{n}\cdot\hat{P}}\right)+\frac{\pi^2}{12}+6\right\},$$

$$C_{4,7}(m_b)=1-\frac{\alpha_s(m_b)C_F}{4\pi}\left\{2\ln^2(\overline{n}\cdot\hat{P})+2\,\mathrm{Li}_2(1-\overline{n}\cdot\hat{P})-\ln(\overline{n}\cdot\hat{P})\left[\frac{2-4\overline{n}\cdot\hat{P}+(\overline{n}\cdot\hat{P})^2}{(1-\overline{n}\cdot\hat{P})^2}\right]+\frac{\overline{n}\cdot\hat{P}}{1-\overline{n}\cdot\hat{P}}+\frac{\pi^2}{12}+6\right\},$$

$$C_{5,8}(m_b)=\frac{\alpha_s(m_b)C_F}{4\pi}\left\{\frac{2}{(1-\overline{n}\cdot\hat{P})}+\frac{2\overline{n}\cdot\hat{P}\ln(\overline{n}\cdot\hat{P})}{(1-\overline{n}\cdot\hat{P})^2}\right\},$$

$$C_9(m_b)=1-\frac{\alpha_s(m_b)C_F}{4\pi}\left\{2\ln^2(\overline{n}\cdot\hat{P})+2\,\mathrm{Li}_2(1-\overline{n}\cdot\hat{P})-2\ln(\overline{n}\cdot\hat{P})+\frac{\pi^2}{12}+6\right\},$$

$$C_{10,11}(m_b)=1-\frac{\alpha_s(m_b)C_F}{4\pi}\left[2\ln^2(\overline{n}\cdot\hat{P})+2\,\mathrm{Li}_2(1-\overline{n}\cdot\hat{P})+\ln(\overline{n}\cdot\hat{P})\left(\frac{4\overline{n}\cdot\hat{P}-2}{1-\overline{n}\cdot\hat{P}}\right)+\frac{\pi^2}{12}+6\right],$$

$$C_{12}(m_b)=0. \tag{33}$$

For the operator $\overline{\xi}_{n,p}\Gamma h_v$ there is only one particle in the jet, so in that case $P=p$. In NDR the relations amongst Wilson coefficients, $C_1=C_2$, $C_3=C_6$, $C_4=C_7$, $C_5=C_8$, $C_{10}=C_{11}$, and $C_{12}=0$ hold true to all orders in perturbation theory for a massless light quark. This is because the transformation, $q\to\gamma_5 q$ is a symmetry of massless QCD and the U(1) helicity symmetry of Eq. (12) allows $\chi_{n,p}\to\gamma_5\chi_{n,p}$, and these transformations relate currents with and without γ_5.

IV. RENORMALIZATION GROUP EVOLUTION

In this section we calculate the running of the Wilson coefficients in the effective theory. The coefficients mix into themselves and satisfy a renormalization group equation of the form

$$\mu\frac{d}{d\mu}C(\mu)=\gamma(\mu)C(\mu). \tag{34}$$

The fact that Eq. (34) is homogeneous reproduces the exponentiation of Sudakov logarithms. In this case it is natural to solve the renormalization group equations for the quantity $\ln C(\mu)$. The leading and subleading series of logarithms are determined by the coefficients summarized in Table II. From the table one can see that for coefficients with tree level

TABLE II. Coefficients in the effective theory loop graphs that we anticipate are needed to predict the series of Sudakov logarithms in $\ln C(\mu)$.

Series in $\ln C(\mu)$		One loop	Two loops	Three loops
LL	$\alpha_s^n \ln^{n+1}$	$1/\epsilon^2$	—	—
NLL	$\alpha_s^n \ln^n$	$1/\epsilon$	$1/\epsilon^2$	—
NNLL	$\alpha_s^n \ln^{n-1}$	matching	$1/\epsilon$	$1/\epsilon^2$

matching, the one-loop matching in Sec. III is not needed until the next-to-next-to-leading logarithmic (NNLL) order.

In Sec. III it was shown that the coefficient of the effective theory current $\bar{\chi}_{n,P}\Gamma h_v$ is the same as the current $\bar{\xi}_{n,p}\Gamma h_v$, so only the renormalization of the simpler $\bar{\xi}_{n,p}\Gamma h_v$ current needs to be considered. At one loop the effective theory diagrams are shown in Fig. 6. To distinguish UV and IR divergences we choose the collinear quark momentum $p=\tilde{p}+k$ with label $\tilde{p}=(\bar{n}\cdot p,0,p_\perp)$ and zero residual momentum $k=0$. In this case $p^2=p_\perp^2\neq 0$ and this off-shellness regulates IR divergences in the diagrams. We will use the Feynman gauge. The soft diagrams in Fig. 6 give

$$\text{Fig. 6(a)}=i\bar{\xi}_{n,p}\Gamma h_v \frac{C_F\alpha_s(\mu)C(\mu)}{4\pi}\bigg[-\frac{1}{\epsilon^2} -\frac{2}{\epsilon}\ln\bigg(\frac{\mu\bar{n}\cdot p}{-p_\perp^2-i\epsilon}\bigg) -2\ln^2\bigg(\frac{\mu\bar{n}\cdot p}{-p_\perp^2-i\epsilon}\bigg)-\frac{3\pi^2}{4}\bigg],$$

$$\text{Fig. 6(b)}=iv\cdot k\frac{\alpha_s(\mu)C_F}{4\pi}\bigg[-\frac{2}{\epsilon}-4 -4\ln\bigg(\frac{\mu}{-2v\cdot k-i\epsilon}\bigg)\bigg], \qquad (35)$$

where k is a residual momentum in the heavy quark wave function diagram. The order λ^0 soft wave function renormalization of the collinear quark is not shown since in Feynman gauge it is proportional to $n^2=0$. Evaluating the diagrams with a collinear gluon in Fig. 6 gives

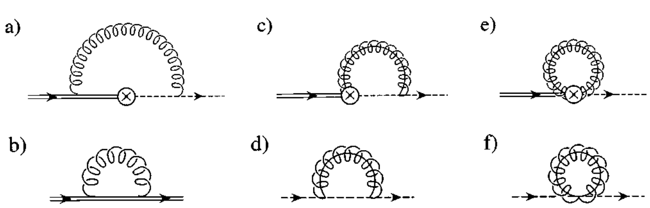


FIG. 6. Order λ^0 effective theory diagrams for the heavy-to-light current at one loop.

$$\text{Fig. 6(c)}=i\bar{\xi}_{n,p}\Gamma h_v\frac{C_F\alpha_s(\mu)C(\mu)}{4\pi}\bigg[\frac{2}{\epsilon^2}+\frac{2}{\epsilon} +\frac{2}{\epsilon}\ln\bigg(\frac{\mu^2}{-p_\perp^2-i\epsilon}\bigg)+\ln^2\bigg(\frac{\mu^2}{-p_\perp^2-i\epsilon}\bigg)+2\ln\bigg(\frac{\mu^2}{-p_\perp^2-i\epsilon}\bigg)+4 -\frac{\pi^2}{6}\bigg],$$

$$\text{Fig. 6(d)}=\frac{i\bar{n}\!\!\!/}{2}\frac{p_\perp^2}{\bar{n}\cdot p}\frac{\alpha_s(\mu)C_F}{4\pi}\bigg[\frac{1}{\epsilon}+1+\ln\bigg(\frac{\mu^2}{-p_\perp^2-i\epsilon}\bigg)\bigg],$$

$$\text{Figs. 6(e), 6(f)}=0. \qquad (36)$$

The graph in Fig. 6(d) was calculated explicitly in Sec. II.

From Eq. (35) we see that the logarithms in diagrams with collinear gluons are small at a scale $\mu\sim\sqrt{p_\perp^2}\sim Q\lambda$. For the graphs with soft gluons the logarithms are small at a different scale $\mu\sim p_\perp^2/(\bar{n}\cdot p)\sim Q\lambda^2$. Running the collinear-soft theory from $\mu=Q$ to $\mu=Q\lambda$ therefore sums all logarithms originating from collinear effects and part of the logarithms from soft exchange. At $\mu=Q\lambda$ collinear gluons may be integrated out and one matches onto a theory containing only soft degrees of freedom. The running in this soft theory includes the remaining logarithms from soft exchange, which would need to be taken into account to sum all Sudakov logarithms.

To run between Q and $Q\lambda$ we add up the ultraviolet divergences in the soft and collinear diagrams in Eqs. (35) and (36). This gives the counterterm in the effective theory

$$Z_i=1+\frac{\alpha_s(\mu)C_F}{4\pi}\bigg[\frac{1}{\epsilon^2}+\frac{2}{\epsilon}\ln\bigg(\frac{\mu}{\bar{n}\cdot P}\bigg)+\frac{5}{2\epsilon}\bigg]. \qquad (37)$$

For $b\to s\gamma$, $\bar{n}\cdot P=m_b$ and Eq. (37) agrees with Ref. [6]. Since $\mu>Q\lambda$ the counterterm can depend on the label $\bar{n}\cdot P\sim Q$, but does not depend on $P_\perp\sim Q\lambda$. Z_i could also have been calculated directly from the matching result in Eq. (31). Since the effective theory reproduces all the infrared divergences in the full theory, the effective theory UV divergences are simply the negative of the full theory IR divergences when pure dimensional regularization is used. This alternative approach also gives Eq. (37).

In the effective theory the current $\bar{\xi}_{n,p}\Gamma h_v$ factors out of the diagrams in Fig. 6 so it is obvious that Z_i is independent of the spin structure of the current. Thus, all the coefficients satisfy the same renormalization group equation (RGE)

$$\mu\frac{d}{d\mu}C_i(\mu)=\gamma(\mu)C_i(\mu). \qquad (38)$$

The LO anomalous dimension is determined by the $\ln(\mu)/\epsilon$ term in Eq. (37) (whose coefficient is determined by the $1/\epsilon^2$ term). The next-to-leading order (NLO) anomalous dimension has a contribution from the $1/\epsilon$ terms in Eq. (37), as well as a contribution from the $\ln(\mu)/\epsilon$ term in the two loop Z_i, counterterm,

$$\gamma_{\rm LO}=-\frac{\alpha_s(\mu)C_F}{\pi}\ln\left(\frac{\mu}{\bar{n}\cdot P}\right),$$

$$\gamma_{\rm NLO}=-\frac{5\alpha_s(\mu)C_F}{4\pi}-2C_FB\frac{\alpha_s^2(\mu)}{(2\pi)^2}\ln\left(\frac{\mu}{\bar{n}\cdot P}\right). \tag{39}$$

We have introduced the notation B for the two loop coefficient that has not yet been computed with the effective theory. From the results in Ref. [9] we are led to expect that $B=C_A(67/18-\pi^2/6)-5n_f/9$.

Since we wish to run down from $\mu=m_b$ and $\bar{n}\cdot P\sim m_b$ it is convenient to introduce the scale m_b into the anomalous dimensions. Writing $\ln(\mu/\bar{n}\cdot p)=\ln(\mu/m_b)-\ln(\bar{n}\cdot\hat{P})$ and noting that the second logarithm is not large, Eq. (39) can be written as

$$\gamma_{\rm LO}=-\frac{\alpha_s(\mu)C_F}{\pi}\ln\left(\frac{\mu}{m_b}\right),$$

$$\gamma_{\rm NLO}=-\frac{\alpha_s(\mu)C_F}{2\pi}\left[\frac{5}{2}-2\ln(\bar{n}\cdot\hat{P})\right]-2C_FB\frac{\alpha_s^2(\mu)}{(2\pi)^2}\ln\left(\frac{\mu}{m_b}\right). \tag{40}$$

Using the one-loop running for $\alpha_s(\mu)$ the LO solution of the RGE is

$$\ln\left[\frac{C_i(\mu)}{C_i(m_b)}\right]=\frac{f_0(z)}{\alpha_s(m_b)}=-\frac{4\pi C_F}{\beta_0^2\alpha_s(m_b)}\left[\frac{1}{z}-1+\ln z\right], \tag{41}$$

where $\beta_0=11/3C_A-2/3n_f$ and

$$z=\frac{\alpha_s(\mu)}{\alpha_s(m_b)}=\frac{2\pi}{2\pi+\beta_0\alpha_s(m_b)\ln(\mu/m_b)}. \tag{42}$$

Equation (41) sums the LL series of Sudakov logarithms between Q and $Q\lambda$. At NLL order we include the $\gamma_{\rm NLO}$ term in the anomalous dimension and the two-loop running of $\alpha_s(\mu)$ in $\gamma_{\rm LO}$ and find the following correction to Eq. (41):

$$\begin{aligned}\ln\left[\frac{C_i(\mu)}{C_i(m_b)}\right]\Bigg|_{\rm NLO}&=f_1(z,\bar{n}\cdot\hat{P})\\&=-\frac{C_F\beta_1}{\beta_0^3}\left[1-z+z\ln z-\frac{1}{2}\ln^2 z\right]\\&\quad+\frac{C_F}{\beta_0}\left[\frac{5}{2}-2\ln(\bar{n}\cdot\hat{P})\right]\ln z\\&\quad-\frac{2C_FB}{\beta_0^2}[z-1-\ln z].\end{aligned} \tag{43}$$

Here $\beta_1=34C_A^2/3-10C_An_f/3-2C_Fn_f$, and z is still given by Eq. (42). It is easy to see that the result in Eq. (43) is suppressed by an extra $\alpha_s(m)$ relative to the result in Eq. (41). Also it is clear that to systematically sum the next-to-leading log series the two loop coefficient B is required.

Combining Eqs. (41) and (43) the final result at a scale $\mu\sim Q\lambda$ is

$$C_i(\mu)=C_i(m_b)\exp\left[\frac{f_0(z)}{\alpha_s(m_b)}+f_1(z)\right]. \tag{44}$$

For $i=\{1,2,3,4,6,7,9,10,11\}$ the matching starts at tree level and from Table II we see that for the LL and NLL solutions the value $C_i(m_b)=1$ should be used in Eq. (44). The coefficients $C_{5,8,12}(m_b)$ are zero at tree level, and inserting their one-loop matching values from Eq. (33) into Eq. (44) gives their LL and NLL series of logarithms.

V. APPLICATIONS

A. Inclusive decays

It is well known that the OPE for heavy-to-light decays converges only for sufficiently inclusive variables. If the available phase space is restricted such that only a few resonances contribute to the decay the assumption of local duality no longer holds and the OPE fails. If, however, phase space is restricted such that highly energetic jets with small invariant mass dominate the decay, only a subset of terms in the OPE are enhanced. It is possible to resum this subset of terms into a universal structure function [20]. In the same region of phase space, large Sudakov logarithms spoil the perturbative expansion and thus have to be summed as well. This summation was carried out for the endpoint of the leptonic energy spectrum in inclusive $B\to X_u l\bar{\nu}$ decays and the endpoint of the photonic energy spectrum in inclusive $B\to X_s\gamma$ decays [9,21] using perturbative factorization [19]. In a subsequent work [10] endpoint logarithms in the hadronic mass spectrum of $B\to X_u l\bar{\nu}$ decays were summed within the same approach. In [6] it was shown that the result for $B\to X_s\gamma$ can be reproduced using the effective field theory.

In this section we consider $B\to X_s\gamma$ and $B\to X_u l\bar{\nu}$ decays at the endpoint of the photonic energy and hadronic invariant mass spectrum, respectively. We define kinematic variables

$$s_0=\frac{p_u^2}{m_b^2},\quad h=\frac{2v\cdot p_u}{m_b}, \tag{45}$$

for $B\to X_u l\bar{\nu}$ decays where p_u is the momentum of the u quark. For $B\to X_s\gamma$ decays we define

$$x=\frac{2v\cdot q}{m_b}. \tag{46}$$

where q is the photon momentum. The endpoint regions mentioned above correspond to $1-x\sim s_0/h\sim\Lambda_{\rm QCD}/m_b$. Thus, the invariant mass of the light jet is of the order $\sqrt{m_b\Lambda_{\rm QCD}}$, and the power counting parameter satisfies $\lambda^2\sim\Lambda_{\rm QCD}/m_b$. At tree level, integrating out collinear modes by performing an OPE in the collinear-soft theory and matching onto soft operators gives

$$\frac{d\hat{\Gamma}_s}{dx} \equiv \frac{1}{\Gamma_s^{(0)}} \frac{d\Gamma_s}{dx} = \langle B|O(x)|B\rangle$$

$$\frac{d^2\hat{\Gamma}_u}{dz\,dh} \equiv \frac{1}{\Gamma_u^{(0)}} \frac{d^2\Gamma_u}{dz\,dh} = 2h^2(3-2h)\langle B|O(z)|B\rangle, \tag{47}$$

where $z=1-s_0/h$. Here we defined the tree level decay rates in the parton model

$$\Gamma_u^{(0)} = \frac{G_F^2}{192\pi^3}|V_{ub}|^2 m_b^5,$$

$$\Gamma_s^{(0)} = \frac{G_F^2}{32\pi^4}|V_{ts}^* V_{tb}|^2 \alpha_{em}[C_7^{full}]^2 m_b^5, \tag{48}$$

where C_7^{full} is the Wilson coefficient of the weak operator mediating the $b\to s$ radiative transition [18] and we neglect contributions from operators other than O_7^{full}. The operator appearing in Eq. (47) is defined as [20]

$$O(y) = \overline{h}_v \delta(i\hat{D}_+ + 1 - y) h_v, \tag{49}$$

where the covariant derivative $\hat{D}_+ = D_+/m_b$ includes only soft gluons. The matrix element of this operator between B-meson states is the light-cone structure function of the B meson. At higher orders in perturbation theory the differential decay rates can be expressed as convolutions of short distance coefficients with the operator $O(y)$. Defining moments of the decay rates

$$\frac{d\hat{\Gamma}_u(N)}{dh} = \frac{1}{\Gamma_u^{(0)}} \int_0^1 dz\, z^{N-1} \frac{d^2\Gamma_u}{dz\,dh},$$

$$\hat{\Gamma}_s(N) = \frac{1}{\Gamma_s^{(0)}} \int_0^1 dx\, x^{N-1} \frac{d\Gamma_s}{dx}, \tag{50}$$

undoes the convolution and makes comparison to existing results in the literature straightforward. Let $\mu_0 \sim Q\lambda$ be the scale where collinear modes are integrated out. At this scale the moments of the decay rates are

$$\frac{d\hat{\Gamma}_u(N)}{dh} = 2h^2(3-2h)C(\mu_0, hm_b)^2\langle O(N;\mu_0)\rangle,$$

$$\hat{\Gamma}_s(N) = C(\mu_0, m_b)^2\langle O(N;\mu_0)\rangle, \tag{51}$$

where the operator $O(N;\mu_0)$ is defined as

$$O(N;\mu_0) = \int_0^1 dy\, y^{N-1} O(y;\mu_0). \tag{52}$$

Various coefficients $C_i(\mu, \overline{n}\cdot p)$ can contribute to the decay rates in Eq. (51). However, at NLL order we only need the tree level matching at $\mu = m_b$ in Eq. (28). Furthermore, at the scale $\mu_0 = m_b/\sqrt{N}$ large logarithms are not introduced when matching onto the operator $O(N,\mu_0)$ [6]. At this scale to NLL order we therefore only need tree level matching onto the operator $O(N, m_b/\sqrt{N})$. Since between $\mu = m_b$ and $\mu = m_b/\sqrt{N}$ all coefficients $C_i(\mu, \overline{n}\cdot p)$ have a universal running, the result can be written in terms of a single coefficient $C(\mu, \overline{n}\cdot p)$, where $C(m_b, \overline{n}\cdot p) = 1$ and runs according to Eq. (44).

The running of $C(\mu, \overline{n}\cdot p)$ does not reproduce the full set of Sudakov logarithms because at $\mu = m_b/\sqrt{N}$ there are additional large logarithms in the matrix element of $O(N,\mu)$. It has been shown that these additional logarithms arise from purely soft gluons and can be summed by running from $\mu = m_b/\sqrt{N}$ to $\mu = m_b/N$ [6]. However, taking the ratio of the decay rates in Eq. (50) these matrix elements cancel,

$$\frac{1}{\hat{\Gamma}_s(N)} \frac{d\hat{\Gamma}_u(N)}{dh} = 2h^2(3-2h)\left[\frac{C(\mu_0, m_b h)}{C(\mu_0, m_b)}\right]^2. \tag{53}$$

Thus, all the Sudakov logarithms in the ratio of rates are calculable from the running of the Wilson coefficients in the collinear-soft theory. Using Eq. (44) this leads to

$$\frac{1}{\hat{\Gamma}_s(N)} \frac{d\hat{\Gamma}_u(N)}{dh} = 2h^2(3-2h)\exp\left[-\frac{4C_F}{\beta_0}\ln(h)\ln(z)\right], \tag{54}$$

where $z(\mu) = \alpha_s(\mu)/\alpha_s(m_b)$ is evaluated at $\mu = m_b/\sqrt{N}$. This result agrees with Ref. [10].

B. Exclusive decays

As another application of the results obtained in Secs. II and III, we investigate exclusive heavy-to-light decays. The nonperturbative physics of such decays is given in terms of form factors. For B decays to pseudoscalar and vector mesons, they are conventionally defined as

$$\langle P(p)|\overline{q}\gamma^\mu b|\overline{B}(p_b)\rangle = f_+(q^2)\left[p_b^\mu + p^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2}q^\mu,$$

$$\langle P(p)|\overline{q}i\sigma^{\mu\nu}q_\nu b|\overline{B}(p_b)\rangle = -\frac{f_T(q^2)}{m_B + m_P}[q^2(p_b^\mu + p^\mu) - (m_B^2 - m_P^2)q^\mu],$$

$$\langle V(p,\epsilon^*)|\overline{q}\gamma^\mu b|\overline{B}(p_b)\rangle = \frac{2V(q^2)}{m_B + m_V} i\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^*(p_b)_\rho p_\sigma,$$

$$\langle V(p,\epsilon^*)|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p_b)\rangle$$
$$=2m_V A_0(q^2)\frac{\epsilon^*\cdot q}{q^2}q^\mu+(m_B+m_V)A_1(q^2)$$
$$\times\left[\epsilon^{*\mu}-\frac{\epsilon^*\cdot q}{q^2}q^\mu\right]-A_2(q^2)\frac{\epsilon^*\cdot q}{m_B+m_V}$$
$$\times\left[p_b^\mu+p^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right],$$

$$\langle V(p,\epsilon^*)|\bar{q}i\sigma^{\mu\nu}q_\nu b|\bar{B}(p_b)\rangle$$
$$=-2T_1(q^2)i\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu(p_b)_\rho p_\sigma,$$

$$\langle V(p,\epsilon^*)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|\bar{B}(p_b)\rangle$$
$$=T_2(q^2)[(m_B^2-m_V^2)\epsilon^{*\mu}-(\epsilon^*\cdot q)(p_b^\mu+p^\mu)]$$
$$+T_3(q^2)(\epsilon^*\cdot q)\left[q^\mu-\frac{q^2}{m_B^2-m_V^2}(p_b^\mu+p^\mu)\right], \tag{55}$$

where $q=p_b-p$. For decays in which the final light meson has large energy, we can use the effective theory to gain additional information on these form factors. Using Eq. (27) the matrix elements in the full theory are given by matrix elements in the effective theory,

$$\langle M|\bar{q}\Gamma b|B\rangle\to\sum_i C_i(\mu)\langle M_{n,P}|\bar{\chi}_{n,P}\Gamma_i h_v|H_v\rangle|_\mu+\Delta F_\Gamma. \tag{56}$$

Here $M=P,V$ corresponds to the light pseudoscalar and vector meson states in the full theory, and $M_{n,P}$ and H_v are the states of the light and the heavy mesons in the effective theory, respectively. The first term in Eq. (56) is the soft contribution, while the second term indicates the so-called hard contributions [22,13]. For the soft form factor the offshellness of the light quark $p_q^2=2Ek_+$ where $k_+\sim\Lambda_{\rm QCD}$, thus $\lambda^2\sim\Lambda_{\rm QCD}/m_b$, just as for the inclusive decays. The ΔF_Γ term in Eq. (56) involves interactions where a collinear gluon is exchanged with the spectator in the B meson. In Ref. [13] it was argued that these spectator effects are the same order in λ and $1/m_b$ as the soft contributions, but can be regarded as being suppressed by a power of $\alpha_s(\sqrt{m_b\Lambda_{\rm QCD}})$. They are therefore just as or more important than the one-loop corrections to the matching coefficients $C_i(\mu)$ given in Eq. (33). Here we will apply the effective theory to the soft contributions and leave the hard spectator contributions for future investigation.

In Ref. [12], Charles *et al.* showed that in heavy-to-light decays, in which both the heavy and light quark interact solely via soft gluons, there are only three independent matrix elements. Charles *et al.* derived their result by combining HQET with LEET and using the fact that the HQET spinors describing heavy quarks h_v and the LEET spinors describing highly energetic quarks interacting with soft gluons, ξ_n, have only two independent components. Using the relations $\not{v}h_v=h_v$ and $\not{n}\xi_n=0$ they showed that at leading order in $1/E$ (where E is the energy of the light meson) matrix elements of all hadronic currents in LEET are determined by only three independent functions. Unfortunately, LEET is not sufficient to describe heavy-to-light decays because it omits interactions with collinear gluons.

However, as pointed out in Sec. II the spinors in the effective theory, describing highly energetic quarks interacting with *both* soft and collinear gluons, still have two components. In Eq. (27) we see that there are only four independent heavy-to-light currents in the collinear soft effective theory. For decays to pseudoscalar mesons like π and K, the matrix elements of these currents are

$$\langle P_n|\bar{\chi}_{n,P}h_v|H_v\rangle=2E\zeta(E),$$
$$\langle P_n|\bar{\chi}_{n,P}\gamma^5 h_v|H_v\rangle=0,$$
$$\langle P_n|\bar{\chi}_{n,P}\gamma_\perp^\mu h_v|H_v\rangle=0, \tag{57}$$

while for decays to vector mesons such as ρ and K^* they are

$$\langle V_n|\bar{\chi}_{n,P}h_v|H_v\rangle=0,$$
$$\langle V_n|\bar{\chi}_{n,P}\gamma^5 h_v|H_v\rangle=-2m_V\zeta_\parallel(E)v\cdot\epsilon^*,$$
$$\langle V_n|\bar{\chi}_{n,P}\gamma_\perp^\mu h_v|H_v\rangle=2E\zeta_\perp(E)i\epsilon_\perp^{\mu\nu}\epsilon^*_\nu, \tag{58}$$

where $\epsilon_\perp^{\mu\nu}=\epsilon^{\mu\nu\sigma\tau}v_\sigma n_\tau$ and we are using relativistic normalization for all effective theory states. Thus, there are still only three linearly independent soft form factors in the complete effective theory. Together with Eq. (27) these matrix elements determine that at tree level the heavy-to-light form factors are

$$f_+(q^2)=\zeta(E),\quad f_0(q^2)=2\hat{E}\zeta(E),\quad f_T(q^2)=\zeta(E),$$
$$A_1(q^2)=2\hat{E}\zeta_\perp(E),\quad A_2(q^2)=\zeta_\perp(E),\quad V(q^2)=\zeta_\perp(E), \tag{59}$$
$$T_1(q^2)=\zeta_\perp(E),\quad T_2(q^2)=2\hat{E}\zeta_\perp(E),\quad T_3(q^2)=\zeta_\perp(E),$$
$$A_0(q^2)=\zeta_\parallel(E).$$

In deriving these relations we have dropped terms suppressed by $m_{P,V}/E$ since these corrections are just as large as λ-suppressed power corrections that are not included. Thus, $\zeta_\parallel(E)$ only appears in the purely longitudinal form factor $A_0(q^2)$. Taking this into account our results are in agreement with Ref. [12].

From the results in Sec. III we can obtain some more information on the heavy-to-light form factors. The results of Eqs. (27) and (33) determine the perturbative corrections to Eq. (59). Hard corrections do not break the symmetry relations between effective theory matrix elements, but do change the relation between form factors in the full and effective theories. We find

$$f_+(q^2)=\zeta(E)[C_4+\hat{E}C_5],$$
$$f_0(q^2)=\zeta(E)2\hat{E}[C_4+C_5(1-\hat{E})],$$

$$\begin{aligned} f_T(q^2) &= \zeta(E) C_{11}, \\ A_1(q^2) &= \zeta_\perp(E) 2\hat{E} C_3, \\ A_2(q^2) &= \zeta_\perp(E) C_3, \\ V(q^2) &= \zeta_\perp(E) C_3, \\ T_1(q^2) &= \zeta_\perp(E) C_9, \\ T_2(q^2) &= \zeta_\perp(E) 2\hat{E} C_9, \\ T_3(q^2) &= \zeta_\perp(E) C_9, \\ A_0(q^2) &= \zeta_\parallel(E)[C_4 + C_5(1-\hat{E})], \end{aligned} \tag{60}$$

where $C_i = C_i(\hat{E})$ and we have used the helicity relations given below Eq. (33). In Ref. [23] it was pointed out that the ratios V/A_1 and T_1/T_2 do not receive perturbative corrections due to the fact that interactions that flip the helicity of the energetic quark are suppressed by $1/E$. From Eq. (60) we see that, in fact, at leading order in λ the soft contributions to the form factors $\{A_1, A_2, V\}$ and $\{T_1, T_2, T_3\}$ are related to all orders in α_s. Furthermore, since the RGE's for all currents are identical, any ratio of soft form factors are independent of Sudakov logarithms.

At one loop the hard corrections to ratios of the form factors in Eq. (55) were previously calculated in Ref. [13]. Since the authors used LEET as their effective theory their matching calculation was not infrared safe and the overall normalization of the low energy matrix elements was unknown. However, it was noted that this problem cancels out of the ratios of form factors because the infrared divergences in the full theory are universal. Our results in Eq. (60) do not suffer from this problem because the collinear-soft effective theory has the same infrared divergences as QCD. Taking ratios of the form factors in Eq. (60), substituting the results in Eq. (33), and expanding in $\alpha_s(m_b)$, our results for the hard corrections to the soft form factors agree with those of Ref. [13].

As an application, consider the zero in the forward-backward asymmetry of the rare decay $B \to K^* l^+ l^-$, which gives a relation between the Wilson coefficients $C_9^{\rm full}$ and $C_7^{\rm full}$ [24,25]

$$\mathrm{Re}\left[\frac{C_9^{\rm full}(s_0)}{C_7^{\rm full}}\right] = -\frac{m_b}{s_0}\left[\frac{T_2(s_0)}{A_1(s_0)}(m_B - m_{K^*}) + \frac{T_1(s_0)}{V(s_0)}(m_B + m_{K^*})\right], \tag{61}$$

where $s_0 \sim 3$ GeV is the value of q^2 where the asymmetry vanishes. It was noted in Ref. [25] that in the ratio of soft form factors the effective theory form factors cancel. Ignoring again the hard spectator contributions and the higher order effect of the mass of the K^* we find

$$\begin{aligned} \mathrm{Re}\left[\frac{C_9^{\rm full}(s_0)}{C_7^{\rm full}}\right] &= -m_B \frac{m_b}{s_0}\left[\frac{2C_9(m_b)}{C_3(m_b)}\right] \\ &= -2m_B \frac{m_b}{s_0}\left[1 + \frac{\alpha_s(m_b) C_F}{4\pi} \times \ln(2\hat{E})\, \frac{2\hat{E}}{1-2\hat{E}}\right], \end{aligned} \tag{62}$$

where the perturbative correction from the soft form factor is in agreement with Ref. [13]. There are additional order α_s corrections to Eq. (62) from collinear gluon exchange with the spectator in B, which can be found in Ref. [13]. Although Sudakov logarithms do not affect the ratio of purely soft form factors, they may suppress the soft contribution relative to that from collinear gluon exchange.

VI. CONCLUSIONS

In this paper we investigated in detail the collinear-soft effective theory, which describes highly energetic particles with low invariant mass. The degrees of freedom in this theory consist of collinear quarks and gluons with momenta scaling as $k_c = Q(\lambda^2, 1, \lambda)$, and soft gluons with momenta scaling as $k_s = Q(\lambda^2, \lambda^2, \lambda^2)$. We gave a detailed derivation of the collinear-soft Lagrangian with the intent of making it straightforward to go to subleading orders in λ. In addition we derived the effective theory heavy-to-light current at order λ^0. For decays of heavy particles there are regions of phase space where this theory applies, namely, when the hadronic decay products are light and are produced with large energy. The currents mediating these decays are given by four linearly independent operators in the effective theory. We performed the matching onto these operators at the one-loop level and calculated their renormalization group evolution from the hard scale $Q \sim m_b$ to the intermediate scale $Q\lambda$.

We considered two applications of the collinear-soft theory: inclusive and exclusive decays. In the inclusive case we focused our attention on the radiative decay $B \to X_s \gamma$ and the semileptonic decay $B \to X_u l \bar{\nu}$ in the endpoint region of large photon energy and of low hadronic invariant mass, respectively. At leading order the OPE in the effective theory gives a bilocal operator whose matrix element is the universal nonperturbative light-cone structure function of the B meson. As is well known, in the ratio of large moments of these two decays, this structure function cancels. As a consequence the Sudakov logarithms in this ratio are entirely determined by the running in the collinear-soft theory as discussed in Sec. V A. Our result is in agreement with previous literature [9,10].

For exclusive decays we investigated the relationship amongst form factors in the large energy limit of QCD. In Ref. [12] it was shown using LEET that there are only three independent soft form factors at leading order in an expansion in inverse powers of the energy of the light quark. However, since LEET does not include collinear gluons it does not correctly reproduce the IR logarithms of QCD, and the relevance of this result is not immediately obvious. We showed that the presence of collinear gluons does not spoil the relations among the soft form factors, therefore establishing these results in the large energy limit of QCD. Finally we used the one-loop matching of the currents in the effective theory to relate the full theory form factors to the three independent matrix elements in the effective theory. Our analysis confirms the corresponding results in Ref. [13], but with an infrared safe definition of the matching coefficients.

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APPENDIX A: COLLINEAR GAUGE TRANSFORMATIONS

In this appendix we discuss the collinear gauge invariance in the soft-collinear effective theory. For simplicity we will restrict ourselves to the Abelian case. From the general set of gauge transformations $U(x)=e^{i\alpha(x)}$, where

$$\psi(x)\to U(x)\psi(x), \quad A_\mu(x)\to A_\mu(x)-\frac{i}{g}U^\dagger(x)\partial_\mu U(x), \tag{A1}$$

the collinear transformations belong to a subset where $\partial_\mu\alpha(x)$ scales like a collinear momentum. To make this scaling explicit we decompose an arbitrary collinear gauge transformation as

$$U(x)\equiv\int d^4Q\, e^{iQ\cdot x}\beta(Q)=\sum_{\tilde{Q}} e^{i\tilde{Q}\cdot x}\beta_Q(x^-), \tag{A2}$$

where $(\bar{n}\cdot Q,Q_\perp,n\cdot Q)\sim(\lambda^0,\lambda,\lambda^2)$ and the sum is over $\tilde{Q}=(\bar{n}\cdot Q,Q_\perp)$. For notational convenience we will suppress the dependence of β_Q on x^- henceforth.

In Sec. II the full quark field was decomposed into components $\xi_{n,p}(x)$ that no longer depend on the large phases $e^{-i\tilde{p}\cdot x}$. Under the collinear gauge transformation in Eq. (A2) we have

$$\sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\xi_{n,p}(x)\to\sum_{\tilde{p}}\sum_{\tilde{Q}} e^{-i(\tilde{p}-\tilde{Q})\cdot x}\beta_Q\xi_{n,p}(x) =\sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\sum_{\tilde{Q}}\beta_Q\xi_{n,p+Q}(x). \tag{A3}$$

Up to terms suppressed by powers of λ the x dependence of $\xi_{n,p}$ can be ignored and the Fourier components must agree, so

$$\xi_{n,p}(x)\to\sum_{\tilde{Q}}\beta_{Q-P}\xi_{n,Q}(x). \tag{A4}$$

Thus, the collinear gauge invariance simply corresponds to a ‘‘reparametrization’’ invariance of the theory under changes to the effective theory labels. Similarly, for the collinear gluon field with label $\tilde{q}$ we find

$$\sum_{\tilde{q}} e^{-i\tilde{q}\cdot x}A^\mu_{n,q}(x)\to\sum_{\tilde{q}} e^{-i\tilde{q}\cdot x}A^\mu_{n,q}(x) +\frac{1}{g}\sum_{\tilde{R}} e^{-i\tilde{R}\cdot x}\sum_{\tilde{Q}}\beta^*_{R+Q} \times[\beta_Q Q^\mu-i\partial^\mu\beta_Q], \tag{A5}$$

so the components transform as

$$A^\mu_q\to A^\mu_q+\frac{1}{g}\sum_{\tilde{Q}}\beta^*_{Q+q}[Q^\mu\beta_Q-i\partial^\mu\beta_Q]. \tag{A6}$$

Using the transformation properties (A4) and (A6) for the collinear quark and gluon field, respectively, it is possible to see that the soft-collinear effective Lagrangian in Eq. (10) is gauge invariant. To see this, it is sufficient to note that the following combination of collinear fields transforms in the same manner as the collinear quark field in Eq. (A3),

$$\sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\left(\tilde{p}^\mu+\frac{\bar{n}^\mu}{2}in\cdot\partial+g\sum_{\tilde{q}} e^{-i\tilde{q}\cdot x}A^\mu_{n,q}\right)\xi_{n,p}. \tag{A7}$$

The derivation is somewhat tedious, so we will not display the details. However, we note that to derive this result it is necessary to make use of the unitarity of the gauge transformation, $U^\dagger(x)U(x)=1$ that implies

$$\sum_{\tilde{P},\tilde{Q}}\beta_Q\beta^*_P e^{i(Q-P)\cdot x}=1. \tag{A8}$$

Finally, we show that the jet field $\chi_{n,P}$ from Sec. III,

$$\chi_{n,P}=\sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\exp\left(\sum_{\tilde{q}} e^{-i\tilde{q}\cdot x}\frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\xi_{n,p}, \tag{A9}$$

is invariant under the collinear gauge transformation in Eq. (A2). We have

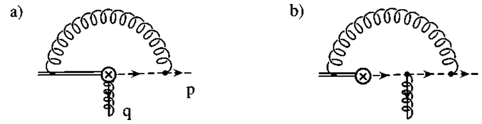

FIG. 7. One-gluon diagrams contributing to the soft renormalization of the current operator $\overline{\chi}_n \Gamma h$ in an Abelian gauge theory. The crossed dot denotes one insertion of the operator $\overline{\chi}_n \Gamma h_v$.

$$\chi_{n,P} \to \sum_{\tilde{p}} e^{-ip\cdot x} \exp\left[\sum_{\tilde{q}} e^{-i\tilde{q}\cdot x} \frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q} + \sum_{\tilde{q}} \frac{e^{-i\tilde{q}\cdot x}}{\bar{n}\cdot q} \sum_{\tilde{Q}} \bar{n}\cdot Q \beta_Q \beta^*_{Q+q} \right] \sum_{R} \beta_{R-p} \xi_{n,R}. \tag{A10}$$

Comparing Eqs. (A1) and (A5) and using $\bar{n}\cdot\partial\beta_Q = 0$ gives

$$-i\bar{n}\cdot\partial\alpha(x) = \sum_{\tilde{R},\tilde{Q}} e^{-i\tilde{R}\cdot x} \beta^*_{R+Q} \beta_Q (-i\bar{n}\cdot Q). \tag{A11}$$

Integrating this result with respect to $n\cdot x/2$ and taking the exponential gives the relation

$$\exp\left[\sum_{\tilde{R},\tilde{Q}} e^{-i\tilde{R}\cdot x} \beta^*_{R+Q} \beta_Q \frac{\bar{n}\cdot Q}{\bar{n}\cdot R} \right] = e^{-i\alpha(x)} = \sum_{\tilde{Q}} e^{-i\tilde{Q}\cdot x} \beta^*_Q. \tag{A12}$$

Substituting Eq. (A12) into Eq. (A10) and shifting $\tilde{p} \to \tilde{R} - \tilde{p}$ leaves

$$\sum_{\tilde{R}} e^{-i\tilde{R}\cdot x} \exp\left[\sum_{\tilde{q}} e^{-iq\cdot x} \frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q} \right] \sum_{\tilde{p}} e^{i\tilde{p}\cdot x} \beta_p \sum_{\tilde{Q}} \times e^{-i\tilde{Q}\cdot x} \beta^*_Q \xi_{n,R} = \chi_{n,P}, \tag{A13}$$

where in the last step we have used the unitarity relation in Eq. (A8) and the definition in Eq. (A9). Thus, the jet field is invariant under a collinear gauge transformation as expected.

APPENDIX B: RENORMALIZATION OF THE CURRENT IN THE COLLINEAR-SOFT THEORY

In Sec. III we quoted the renormalization constant of the current operator $\overline{\chi}_{n,P}\Gamma h$, in the collinear-soft theory

$$Z = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\log\frac{\mu}{\bar{n}\cdot P} + \frac{5}{2\epsilon} \right). \tag{B1}$$

This was obtained by computing the renormalization of the term $\bar{\xi}_{n,p}\Gamma h_v$, which is the first term obtained using the expansion of $\overline{\chi}_{n,P}$ in Eq. (25). However, the renormalization constant (B1) depends only on the large component of the jet momentum $\bar{n}\cdot P$, which enters as a label on the jet field $\chi_{n,P}$. This is a nontrivial consequence of collinear gauge invariance and is essential for a consistent renormalization of the collinear-soft effective theory. For example, in the collinear diagram in Fig. 6(c) the Wilson coefficient depends on only the sum of the collinear gluon and quark momentum in the loop. Thus, it depends only on p and not on the loop momentum.

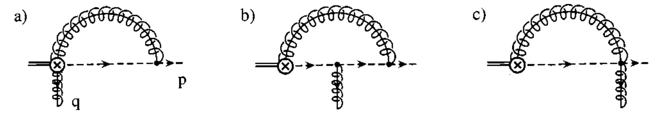

FIG. 8. Collinear gluon renormalization of the current operator $\overline{\chi}_n \Gamma h$ with one external collinear gluon. The crossed dot denotes one insertion of the operator $\overline{\chi}_n \Gamma h_v$.

In this appendix we illustrate this property of the current by explicit calculation of the corresponding renormalization of the one collinear gluon term in the expansion of $\overline{\chi}_{n,P}\Gamma h_v$. For simplicity we will work with an Abelian gauge theory (QED), for which this expansion has been given explicitly in Eq. (21) (with $P = p + \Sigma_i q_i$ for each term),

$$\overline{\chi}_{n,P}\Gamma h_v = \bar{\xi}_{n,P}\Gamma h_v - \frac{g}{\bar{n}\cdot q}\bar{\xi}_{n,P}\bar{n}\cdot A_{n,q}\Gamma h_v + \frac{g^2}{\bar{n}\cdot q_1 \bar{n}\cdot q_2}\bar{\xi}_{n,p}\bar{n}\cdot A_{n,q_1}\bar{n}\cdot A_{n,q_2}\Gamma h_v + \cdots. \tag{B2}$$

The diagrams contributing to the renormalization of the second term in this expansion are shown in Figs. 7 and 8. We will work throughout in the Feynman gauge. The diagrams in Figs. 7(a) and 8(a) have been computed already; the external gluon momentum q does not enter the loop integral, so they can be simply extracted from the corresponding results for $\bar{\xi}_{n,P}\Gamma h_v$ [Eqs. (35), (36)],

$$\text{Figs. 7(a)+8(a)} = \left\langle -\frac{g}{\bar{n}\cdot q}\bar{\xi}_{n,p}\bar{n}\cdot A_{n,q}\Gamma h_v \right\rangle \frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \frac{2}{\epsilon}\log\frac{\mu}{\bar{n}\cdot p} + \text{const} \right). \tag{B3}$$

Furthermore, upon examining the Feynman rule for the two collinear gluon coupling in Fig. 1, one can see that the graph in Fig. 8(c) vanishes in the Feynman gauge. We will show in the following that the net effect of the two remaining graphs Figs. 7(b) and 8(b) is to change $\bar{n}\cdot p$ in the argument of the logarithm in Eq. (B3) to $\bar{n}\cdot(p+q)$, corresponding to the total momentum $P = p + q$ carried by the jet.

For simplicity we will take the external momenta p,q to be off-shell and to have vanishing transverse components, $p = (p_+, p_-, 0_\perp)$ and $q = (q_+, q_-, 0_\perp)$. With this choice the soft diagram Fig. 7(b) reduces to one term,

$$\text{Fig. 7(b)} = -ig^3 \int \frac{d^d l}{(2\pi)^d} \frac{\langle \bar{\xi}_{n,p} n\cdot A_{n,q} \Gamma h_v \rangle \mu^{2\epsilon}}{[n\cdot l - p^2/\bar{n}\cdot p][n\cdot l - (p+q)^2/\bar{n}\cdot(p+q)][v\cdot l][l^2]}. \tag{B4}$$

The integration is performed most easily in light-cone coordinates $l=(l^+,l^-,l_\perp)$, where the l^+ integral can be done by the method of residues. We obtain

$$\text{Fig. 7(b)} = \langle \bar{\xi}_{n,p} n\cdot A_{n,q}\Gamma h_v\rangle \frac{g^3}{8\pi^2}\frac{1}{n\cdot q}\left\{\frac{1}{\epsilon}\ln\left[\frac{n\cdot(p+q)}{n\cdot p}\right] + \text{const}\right\}. \tag{B5}$$

Since this graph does not give a contribution to $\langle \bar{\xi}_{n,p} \bar{n}\cdot A_{n,q}\Gamma h_v\rangle$, it does not contribute to the renormalization of the current. However, the resulting divergence seems to require a new truly nonlocal operator. We will show that this contribution cancels in the sum of diagrams.

The collinear graph in Fig. 8(b) can be written as

$$\text{Fig. 8(b)} = \langle \bar{\xi}_{n,p} n\cdot A_{n,q}\Gamma h_v\rangle I_1 + \langle \bar{\xi}_{n,p}\bar{n}\cdot A_{n,q}\Gamma h_v\rangle I_2, \tag{B6}$$

where

$$I_1 = 2ig^3\mu^{2\epsilon}\int \frac{d^d l}{(2\pi)} \frac{\bar{n}\cdot(p+l)\bar{n}\cdot(p+q+l)}{\bar{n}\cdot l(l+p)^2(l+p+q)^2 l^2}, \tag{B7}$$

$$I_2 = -2ig^3\mu^{2\epsilon}\int \frac{d^d l}{(2\pi)^d} \frac{l_\perp^2}{\bar{n}\cdot l(l+p)^2(l+p+q)^2 l^2}. \tag{B8}$$

Once again the integration is simplified by using the method of residues on the l^+ integral. Explicitly, we find

$$I_1 = -\frac{g^3}{8\pi^2}\frac{1}{n\cdot q}\left\{\frac{1}{\epsilon}\ln\left[\frac{n\cdot(p+q)}{n\cdot p}\right] + \text{const}\right\}, \tag{B9}$$

$$I_2 = -\frac{g}{\bar{n}\cdot q}\frac{\alpha_s}{4\pi}\left\{\frac{2}{\epsilon}\ln\left[\frac{\bar{n}\cdot p}{\bar{n}\cdot(p+q)}\right] + \text{const}\right\}. \tag{B10}$$

Thus, the first term in I_1 cancels the UV divergence in Eq. (B5), as required. The divergent term in I_2 converts the label in Eq. (B3) from $\bar{n}\cdot p$ to $\bar{n}\cdot(p+q)$. As mentioned, the remaining UV divergence depends only on the total jet momentum $P=p+q$,

$$\text{Figs. 7(a)+7(b)+8(a)+8(b)+8(c)} = \left\langle -\frac{g}{\bar{n}\cdot q}\bar{\xi}_{n,p}\bar{n}\cdot A_{n,q}\Gamma h_v\right\rangle \frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \frac{2}{\epsilon}\log\frac{\mu}{\bar{n}\cdot P} + \text{const}\right). \tag{B11}$$

After adding the contributions from the heavy quark and collinear quark field wave function renormalization, we reproduce the renormalization constant Z in Eq. (B1) (after taking the color factor $C_F\to 1$). With similar techniques we have also checked that this holds for the renormalization of the term in Eq. (B2) that contains two collinear gluon fields. As argued in Sec. III, collinear gauge invariance forces all the terms in the sum in Eq. (B2) to be renormalized in the same way, with a Wilson coefficient that depends only on the total jet momentum. The explicit calculations in this appendix agree with this result.

[1] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[2] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); H. Georgi, *ibid.* **240**, 447 (1990).
[3] M. A. Shifman and M. B. Voloshin, Yad. Fiz. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)]; J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B **247**, 399 (1990); I. I. Bigi, M. Shifman, N. G. Uraltsev, and A. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).
[4] M. J. Dugan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).
[5] U. Aglietti, G. Corbo, and L. Trentadue, Int. J. Mod. Phys. A **14**, 1769 (1999); C. Balzereit, T. Mannel, and W. Kilian, Phys. Rev. D **58**, 114029 (1998).
[6] C. W. Bauer, S. Fleming, and M. Luke, Phys. Rev. D **63**, 014006 (2001).
[7] M. E. Luke, A. V. Manohar, and I. Z. Rothstein, Phys. Rev. D **61**, 074025 (2000).
[8] V. A. Smirnov, Phys. Lett. B **404**, 101 (1997); M. Beneke and V. A. Smirnov, Nucl. Phys. **B522**, 321 (1998); V. A. Smirnov and E. R. Rakhmetov, Theor. Math. Phys. **120**, 870 (1999); V. A. Smirnov, Phys. Lett. B **465**, 226 (1999); for applications see J. H. Kuhn, A. A. Penin, and V. A. Smirnov, Eur. Phys. J. C **17**, 97 (2000); M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. **B591**, 313 (2000).
[9] R. Akhoury and I. Z. Rothstein, Phys. Rev. D **54**, 2349 (1996); G. P. Korchemsky and G. Sterman, Phys. Lett. B **340**, 96 (1994).
[10] A. K. Leibovich, I. Low, and I. Z. Rothstein, Phys. Rev. D **62**, 014010 (2000).
[11] V. Chernyak and I. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990).
[12] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999).
[13] M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2000).
[14] S. J. Brodsky, H. Pauli, and S. S. Pinsky, Phys. Rep. **301**, 299 (1998); R. Venugopalan, nucl-th/9808023; H. Leutwyler, Nucl. Phys. **B76**, 413 (1974).
[15] J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D **3**, 1382 (1971); J. B. Kogut and D. E. Soper, *ibid.* **1**, 2901 (1970).

[16] M. Luke and A. V. Manohar, Phys. Rev. D **55**, 4129 (1997); M. Luke and M. J. Savage, *ibid.* **57**, 413 (1998).
[17] W. Zhang and A. Harindranath, Phys. Rev. D **48**, 4881 (1993).
[18] B. Grinstein, R. Springer, and M. B. Wise, Phys. Lett. B **202**, 138 (1988); B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).
[19] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981); J. C. Collins, D. E. Soper, and G. Sterman, *ibid.* **B261**, 104 (1985); See J. C. Collins, in *Perturbative Quantum Chromodynamics*, Advanced Series on High Energy Physics Vol. 5, edited by A. H. Mueller (World Scientific, Singapore, 1989), and references therein.
[20] I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994); M. Neubert, Phys. Rev. D **49**, 4623 (1994).
[21] A. K. Leibovich and I. Z. Rothstein, Phys. Rev. D **61**, 074006 (2000); A. K. Leibovich, I. Low, and I. Z. Rothstein, *ibid.* **61**, 053006 (2000).
[22] R. Akhoury, G. Sterman, and Y. P. Yao, Phys. Rev. D **50**, 358 (1994).
[23] G. Burdman and G. Hiller, hep-ph/0011266.
[24] A. Ali, T. Mannel, and T. Morozumi, Phys. Lett. B **273**, 505 (1991).
[25] G. Burdman, Phys. Rev. D **57**, 4254 (1998); A. Ali, P. Ball, L. T. Handoko, and G. Hiller, *ibid.* **61**, 074024 (2000).